Handbook of Research on Geoinformatics

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Chapter XII
Overview, Classification and Selection of Map Projections for Geospatial Applications

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ABSTRACT

There has been a dramatic increase in the handling of geospatial information, and also in the production of maps. However, because the Earth is three-dimensional, geo-referenced data must be projected on a two-dimensional surface. Depending on the area being mapped, the projection process generates a varying amount of distortion, especially for continental and world maps. Geospatial users have a wide variety of projections to choose from; it is therefore important to understand distortion characteristics for each of them. This chapter reviews foundations of map projection, such as map projection families, distortion characteristics (areal, angular, shape and distance), geometric features and special properties. The chapter ends by a discussion on projection selection and current research trends.

INTRODUCTION

Recent automation and increasing user-friendliness of geospatial systems (such as Geographical Information Systems -GIS) has made the production of maps easier, faster and more accurate. Cartographers have at present an impressive number of projections, but often lack a suitable classification and selection scheme for them, which significantly slow down the mapping process. Map projections generate distortion from the original shape of the Earth. They distort angles between locations on Earth, continental areas and distances between points. Distortion, although less apparent on a larger-scale map (because it covers a smaller area and the curvature of the Earth is less pronounced), misleads people in the way they visualize, cognize or locate large geographic features (Snyder, 1993). Map projections have been devised to answer some of the distortion issues, preserving selected geometric properties (e.g., conformality, equivalence, and equidistance) and special properties. Ignoring these distortion characteristics may lead to an unconsidered choice of projection framework,
resulting in a disastrous map, thereby devaluing the message the map attempts to communicate. It is urgent for users of geospatial technologies to acquire a map projection expertise before interacting with any cartographic software.

THE MAP PROJECTION PROCESS

The Earth is essentially spherical, but is approximated by a mathematical figure—a datum surface. For the purpose of world maps, a sphere with radius \( R_E = 6371 \text{km} \) is a satisfying approximation. For large-scale maps however (i.e., at the continental and country scale), the non-spherical shape of the Earth is represented by an ellipsoid with major axis \( a \) and minor axis \( b \). The values of \( a \) and \( b \) vary with the location of the area to be mapped and are calculated in such a way that the ellipsoid fits the geoid almost perfectly. The full sized sphere is greatly reduced to an exact model called the generating globe (see Figure 1). Nevertheless, globes have many practical drawbacks: they are difficult to reproduce, cumbersome for measuring distances, and less than the globe is visible at once.

Those disadvantages are eliminated during the map projection process, by converting the longitude and latitude angles \((\lambda, \phi)\) to Cartesian coordinates (Canets and Decleir, 1989):

\[
x = f(\phi, \lambda), \quad y = g(\phi, \lambda)
\]

CLASSIFICATION OF MAP PROJECTIONS BY FAMILIES

Three major projections classes are named after the developable surface onto which most of the map projections are at least partially geometrically projected. All three have either a linear or punctual contact with the sphere: they are the cylindrical, conical and azimuthal. The advantage of these shapes is that, because their curvature is in one dimension only, they can be flattened to a plane without any further distortion (Iliffe, 2000). The pseudocylindrical, pseudoconic, pseudoazimuthal and pseudoconical projections are based on the three aforementioned families (Snyder, 1987; Lee, 1944).

Conical Projection

When a cone wrapped around the globe is cut along a meridian, a conical projection results. The cone has its peak—also called apex—above one of the two Earth’s poles and touches the sphere along one parallel of latitude (Figure 2a). When unwrapped, meridians become straight lines converging to the apex, and parallels are represented by arcs of circle. The pole is either represented as a point or as a line. When the cone is secant to the globe, it bisects the surface at two lines of latitude (Figure 2b).

Cylindrical Projection

A cylinder is wrapped around the generating globe, so that its surface touches the Equator throughout its circumference. The meridians of longitude are of equal length and perpendicular to the Equator. The parallels of latitude are marked off as lines

Figure 1. The map projection process: the sphere, approximated by a mathematical figure is reduced to a generating globe that is projected on a flat surface. (after Canets and Decleir 1989)
parallel to the Equator, and spaced in such a way to preserve specific properties, described further. The final process consists of cutting the cylinder along a specific meridian yielding a cylindrical map (Figure 2c). An example of a tangent projection is the Plate Carrée projection with transformation formulas $x = R(\lambda - \lambda_0)$ and $y = R\phi$, where $\lambda_0$ is the longitude of the prime meridian. A secant cylindrical projection is obtained when the cylinder bisects the globe at two parallels (Figure 2d). The Behrman projection is a secant projection characterized by standard parallels at $\pm 30^\circ$ with transformation formulas $x = R(\lambda - \lambda_0)\cos\phi_0$, $y = R\phi$, where $\phi_0$ denotes the standard latitude.

Azimuthal Projection

An azimuthal projection results from the projection of meridians and parallels at a point on a plane tangent on one of the Earth’s poles. The meridians become straight lines diverging from the center of the projection. The parallels are portrayed as concentric circles, centered on the chosen pole. Azimuthal projections are widely used to portray the polar areas (Figure 3). Three different kinds of azimuthal projection are possible:

Figure 2. Illustration of the tangent conical projection in (a) and a secant projection in (b). Illustration of the tangent cylindrical projection in (c) and its secant counterpart in (d). Distortion is minimum on the contact lines and increases away from those parallels of latitude.

Figure 3. Illustration of the azimuthal projection and the three resulting cases

Orthographic  Stereographic  Gnomonic
1. The orthographic case results from a perspective projection from an infinite distance, which gives a globe-like shape. Only one hemisphere can be mapped at a time, and distortions are greatest along the rim of the hemisphere, where distances and landmasses tend to be compressed.

2. The stereographic case is a true perspective projection with the globe being projected onto the plane from the point on the globe diametrically opposite to the point of tangency, also called nadir. The stereographic case preserves angles.

3. A gnomonic projection is obtained by projection on the plane from the center of the globe. The shape of the countries is dramatically distorted away from the center of the plane. Straight lines on a gnomonic projection depict great circles. Unfortunately, the gnomonic projection excessively distorts shapes. Its use is recommended for comparison with the Mercator and plotting long-distance courses of ships and airplanes (see Figure 4).

Pseudoconic and Polyconic Projections

The pseudoconic projection, such as the Boone’s equal-area, has its parallels represented as concentric circular arcs, and meridians as concurrent curves. A polyconic projection results from projecting the Earth on different cones tangent to each parallel of latitude. Both meridians are parallels represented by concurrent curves. The Van Der Grinten and Lagrange are polyconic projections. (Snyder and Voxland, 1994).

Pseudocylindrical Projection

This projection is characterized by straight lines of latitude and concurrent curved meridians. Robinson’s projection, adopted by the National Geographic Society is a key example (Robinson, 1974), and so is the sinusoidal equal-area (Figure 5).

DISTORTION GENERATED DURING THE PROJECTION PROCESS

Projections generate distortion from the original shape of the globe by shearing, compression and tearing of continental areas. On world maps, continental areas may severely be altered, increasingly away from the central meridian.

Distortion Indexes

The scale distortion \( m \) is defined by the ratio of a projected length \( ds \) determined by two points over the original length \( DS \) on the generation globe. The

Figure 4. From left to right: the Mercator projection centered 60°N, 35°W, and the gnomonic projection centered 60°N, 47°W. Great circles from Oslo to Anchorage and San Diego are represented as straight lines on the gnomonic projection and are curved on the Mercator projection.
terms $h$ and $k$ denote the scale distortion along the meridian and parallels, respectively (Canters and Decler, 1989):

$$m = \frac{ds}{DS}, \quad h = \frac{ds_m}{DS_m} = \frac{\sqrt{E}}{R}, \quad k = \frac{dp}{DS_p} = \frac{\sqrt{G}}{R \cos \varphi}$$

(2)

$$m = \sqrt{\frac{Ed\varphi^2 + 2Fd\varphi d\lambda + Gd\lambda^2}{(Rd\varphi)^2 + (R \cos \varphi d\lambda)^2}}$$

(3)

$E$, $F$ and $G$ are further defined as:

$$F = \left( \frac{\partial x}{\partial \varphi} \right) \left( \frac{\partial x}{\partial \lambda} \right) + \left( \frac{\partial y}{\partial \varphi} \right) \left( \frac{\partial y}{\partial \lambda} \right) \quad G = \left( \frac{\partial x}{\partial \lambda} \right)^2 + \left( \frac{\partial y}{\partial \lambda} \right)^2$$

(4, 5, 6)

The term $m$ is equal to 1 everywhere on the globe, however, $m$ cannot be equal to 1 on the map except along specific lines (contact lines) or at center points where the distortion is inexistential. The scale distortion $m$ varies from point to point and fluctuates in every direction. When the developable surface has only one point/line of contact with the sphere, the value of $m$ will increase away from that point or line. When the developable surface cuts the globe, the area between the two standard lines is reduced ($m<1$) and stretched ($m>1$) away from the contact lines. When $F$ is made equal to zero, the projection is said to be orthogonal, which means that parallels and meridians form a perpendicular network, such as on cylindrical projections.

Angular Distortion

A conformal projection gives the relative local directions correctly at any given point. It is obtained when the scale distortion is independent from azimuth or is the same in every direction. $F$ is made equal to zero, and $h=k$ all over the map, however not equal to 1.

$$\frac{E}{R^2} = \frac{G}{R^2 \cos^2 \varphi}$$

(10)

Distance Distortion

Constant scale cannot be maintained throughout the whole map. An equidistant projection shows the length of either all parallels or meridians correctly. To obtain an equidistant projection along the parallels, $k$ must be equal to 1 everywhere. An equidistant projection that shows $h=1$ preserves the length of the meridians. This is often the case for cylindrical projections. In no cases can $h$ be equal to $k$ and be equal to 1, except for standard line(s)/point.

Figure 5. Tissot's indicatrices on the Sinusoidal equal-area projection to the left and the oblique conformal Mercator. The indicatrices remain circles on the oblique Mercator's projection since it preserves angles ($a=b$ everywhere). Nevertheless, their sizes increase away from the centerline, causing an increase of the scale distortion (the areal distortion for Africa and Alaska becomes excessive). On the Sinusoidal projection however, the area of the indicatrices remains the same, but their orientation varies.
Areal Distortion

A map is said equal-area when the representation of surface on the sphere is equal to that surface on the map, and hence represented in correct relative size:

\[ DS_m DS_p = ds_m ds_p \sin \theta' \]  \hspace{1cm} (11)

where on the projected map \( \theta' \) denotes the angle between the parallels and meridians on the map, and \( ds_m \) and \( ds_p \) represent an infinitesimal length of a meridian and a parallel respectively. The term \( DS_m DS_p \) can be calculated on the sphere as \( R^2 \cos \phi \, d\phi \, d\lambda \). \( DS_m \) and \( DS_p \) are the corresponding distances on the globe (Canters and Decleir, 1989).

The Indicatrix of Tissot

Tissot (1881) proved that it is not possible to combine all geometric properties (conformality, equivalence and equidistance) together in a single projection. He studied the distortion of infinitesimally small circles on the surface of the Earth, for which he plotted the values of \( m \) in all directions. The resulting geometric figure is called Tissot’s indicatrix. The angle formed by the intersection of two lines on the Earth can be represented on the final map either by the same angle or not. It the angle is similar, the projection is conformal. Tissot demonstrated he could find two lines in every point of the Earth that, after the transformation process, will remain perpendicular on the map. Along these two directions occur the minimum and maximum distortion. The major axis \( a \) is the direction of maximal distortion, while the minor axis \( b \) is the direction of minimal distortion (this value can be less than 1, which results in a compression):

\[
\begin{align*}
    a &= \left| \frac{ds}{D_s} \right|_x, \\
    b &= \left| \frac{ds}{D_s} \right|_y
\end{align*}
\]  \hspace{1cm} (11)

From what has been discussed before, a relation among \( h, k, a \) and \( b \) can be obtained:

\[ h^2 + k^2 = a^2 + b^2 \]  \hspace{1cm} (12)

Note that \( ab=1 \) and \( a=b \) are two mutually exclusive properties, yet \( a=b=1=ab \) on the standard lines. A projection can never be equal-area and conformal at the same time. Tissot’s theory gives a general perception of the distortion of the projection (Figure 5).

GEOMETRIC FEATURES

Spacing of the Parallels

On most projections, the spacing of the parallels highly influences the preservation of the equal-area property (Hsu, 1981). An equal spacing of the parallels avoids extreme compression or stretching in the North-South direction. A decreasing spacing is often the guarantee to meet this criterion, at the cost of a severe compression of the polar areas. A stronger convergence of the meridians towards the poles can prevent this compression, at the cost of a higher angular distortion.

The Aspect of the Map

Although the construction’s principles remain unchanged, the above developable surfaces can be oriented differently. The aspect of a projection refers to the angle formed by the axis of the cylinder/cone and the Earth’s axis (Snyder, 1987). The angle can be between these two extreme values and resulting in an oblique projection, whereby meridians and parallels are not straight lines or arcs of circle anymore.

The Choice of the Central Meridian

The choice of a central meridian (also called prime meridian) can be very critical. In general, the projection is centered on Greenwich, which gives a European viewpoint. Its location impacts people’s mental map of the world (Saarinen, 1999). On a cylindrical projection, the choice of a central meridian is not so relevant, yet it is very critical on a pseudocylindrical, pseudoconic or polyconic projection since the continental areas located at the outer edges of the map are more distorted.
Overview, Classification and Selection of Map Projections for Geospatial Applications

The Outline of the Map

The outline of the map influences the message the map communicates. A circular outline gives an impression of the spherical shape of the Earth (Dahlberg, 1991). A rectangular outline has the advantage that it fits well in an atlas. Many critics have risen against the use of rectangular map projections, especially the Mercator’s and Peters’ projections. Robinson (1988) and the American Cartographic Association (1989) stress the misconceptions generated by rectangular grids: the Earth is not a square; it is thus essential to choose a world map that portrays the roundness of the world better.

Representation of the Poles

The Poles can be represented as a line or as a point (pointed-polar projection). The first has the inconvenience of an unacceptable stretching in the polar areas along the E-W axis, while the latter compresses the northern landmasses (Canter, 2002). Compromise projections such as pseudocylindrical projections can prevent this.

Ratio of the Axes

Preserving a correct ratio of the axes of the projection prevents an extreme stretching of the map and leads to a more balanced distortion pattern. A correct ratio (2:1) presumes a length of the equator twice the length of the central meridian, yielding to a pleasing map.

Continuity

The property of continuity, i.e., that the projection forms a continuous map of the whole world, is important in maintaining the concept that the Earth has no edges and that the study of the relationship of world distributions should not be confined by the artificial boundary of the map. This is very relevant for mapping continuous purposes, such as climatic phenomena (Wong, 1965).

Correct Azimuth

A projection showing azimuths correctly is an important feature in navigational charts and has important applications in representing radar ranges for instance (Hsu, 1981). On azimuthal projections, all great circles that pass through the center of the projection will be represented as straight lines radiating from the center of the projection.

SELECTION OF SUITABLE MAP PROJECTIONS FOR GEOSPATIAL APPLICATIONS

A projection, when well chosen, could maximize the communication of the map. Too often, a projection is rapidly chosen because it is the first-at-hand (Hsu, 1981). A quantitative analysis of the deformation on a projection would help to retard the tendency towards the selection of a too conventional map projection (Robinson 1951).

The map projection selection process is very challenging: the geospatial user has to choose among an abundant variety of projections, determined by the software in use. For the sole purpose of world-maps, most cylindrical projection should be disregarded and replaced by more appropriate projections, except when straight meridians are required (see Table 1 for a non-exhaustive list of popular projections). Generally, pseudocylindrical and polyconic projections are preferred. The use of a minimum-error map projection is advised for general-purpose mapping, since it results in a map that better fulfills the constraints imposed and guarantees a minimum visual distortion (Canter 2002). These projections are very acceptable from a quantitative and perceptive point of view.

Data Transfer in GIS Among Supported Projections

The combination of geospatial data sets from one projection framework with those from another can hamper the visual display of the geographic features (Goodchild, 1991). Geospatial data is often
collected in different projections. Geographic data \((\lambda, \phi)\) is commonly displayed on a Platte Carrée projection. However, this data can be plotted in a new projection, more suitable for his/her final purpose:

\[
(x_a, y_a) \rightarrow (\phi, \lambda) \rightarrow (x_b, y_b)
\]  

(13)

where \(x_a\) and \(y_a\) are the Cartesian coordinates of the original projection, \(x_b\) and \(y_b\) the coordinates of the final projection. The conversion from geographical coordinates to Cartesian coordinates is the normal process and is regarded as the forward solution. The inverse solution is the preliminary conversion required to find the geographical coordinates from the original Cartesian coordinates \(x_a\) and \(y_a\) (Maling, 1991). Note that the original coordinates may have been digitized, and then are converted in a final common projection framework. If the data is recorded in geographic coordinates \(\lambda\) and \(\phi\), the inverse solution is not needed. The user should be aware that the ellipsoid system may have been used to record his or her data. Data sets displayed in the same projections but measured on two different geodetic systems will not be displayed properly. However, current GISs support conversion among different geodetic reference systems.

**Map Projection Selection**

The purpose of the map specifies the properties that the map projection must have and therefore limits the set of candidate projections (Gilmartin, 1985, Maling, 1992). Snyder (1987) and Iliffe (2000) discuss criteria to decide upon a suitable projection:

- the projection should preserve any properties that the use of the map dictates; and
- additional geometric properties should be considered after the scale factor.

The size and the location of the area to be displayed affect the projection decided upon. A suitable projection is one that minimizes the scale factor over the region.

The need for special properties should be considered for maps of areas larger or equal to a hemisphere; these are the preservation of angles, areas, distances, straight loxodrome or minimal distortion. When the mapmaker deals with smaller areas, the selection is primarily based on the extent of the region unless a special property is required that limits the choice to just one projection (Kessler, 1992).

Maling (1992) defined three rules that should be taken into consideration when determining the projection class:

- Azimuthal projection must be used for maps of the polar regions;
- Conical projections are to be preferred for areas of middle latitudes; and
- Equatorial regions are best mapped using cylindrical projections.

If transverse and oblique aspects are also to be taken into consideration (they might greatly reduce the distortion when applied properly), these rules can be put aside and the projection class is a function of the shape defined by the region, regardless of its geographical location.

**Considerations for an Objective Approach**

The purpose of the map defines whether any special properties are required. A map showing statistical data requires an equivalent projection, whereas conformal projections are preferred if accurate angles of flows are greatly needed. Geometric properties will by themselves narrow the number of candidate projections. These properties have a visual influence on the look of the map and consequently a lasting/pleasing effect to the eye. It should be noted that even if an equivalent projection is required, a compromise or minimum-error projection could sometimes better portray the continental shapes while preserving the message of the map.

**FUTURE RESEARCH**

Most of the present research has focused on constructing new, optimal projection for the given data and problem (Laskowski, 1997). Canters (2002) for instance developed tailor-made maps, either by
modifying existing projections, or by creating new ones minimizing overall distortion and preserving specific properties. Current research is performed in the development of distortion measures to quantify the joint contribution of shape distortion and relative distortion of area, so that both the shape and the relative size of the individual landmasses are well represented (Canters, 2005).

REFERENCES


**KEY TERMS**

**Cartesian Coordinate System:** A plane coordinate system based on a map projection, where the location of a feature is determined by $x$ and $y$.

**Central Meridian:** An imaginary meridian that serves as the starting point for measuring longitude. Passes through the Royal Observatory in Greenwich, England

**Ellipsoid:** A model that approximated the shape of the Earth. Also called spheroid.

**Generating Globe:** Also called reference globe, it is a reduced model of the Earth, from which map projections are made.

**Geographic Coordinate System:** A location reference system for spatial features on the Earth, where the location of a geographic feature is determined by angles $\lambda$ and $\phi$.

**Great Circle:** An imaginary circle made on the Earth’s surface by a plane passing through its center. It is also the shortest distance between two points on the Earth. Represented by straight on a gnomonic projection.

**Inverse Coordinate Transform:**

**Latitude:** The angle between the point and the equator along a meridian (Figure 1)

**Longitude:** The angle on the equatorial plane between the meridian of the point and the central meridian (through Greenwich, England)

**Meridians of Longitude:** Denoted $\lambda$, meridians connect the North Pole to the South poles by a set of imaginary lines, and perpendicular with each parallels of latitude. The length of $1^\circ$ of longitude varies with varying latitude. At the Equator, they are the same length as a degree of latitude, but decrease polewards (Snyder, 1987)

**Parallel of Latitude:** Denoted $\phi$, a parallel is formed by circles surrounding the Earth and parallel to the Equator. Parallels of latitude are drawn equally spaced within the $90^\circ$ separation between the poles and the Equator. The circles are numbered from $0^\circ$ at the Equator to $90^\circ$ at the poles. The radius of parallel decreases polewards at a rate of $R \cos \phi$.

**Orthophanic:** A projection is said orthophanic when it is right-appearing. The Robinson projection, is an example. Such projections are pleasing to the eye, especially because the distortion of continental shapes remains acceptable from a perceptive point of view (McLeary, 1989).

**Rhumb Line:** Lines of constant bearing (direction). Represented as a straight line on the Mercator projection, but as a curve on the gnomonic.

**Transformation Formula:** Mathematical formula allowing the transfer of data from geographic coordinates to Cartesian coordinates.