Synthesis of infrared spectral signatures

Kenneth J. Barnard, MEMBER SPIE
Glenn D. Boreman, MEMBER SPIE
University of Central Florida
Electrical Engineering Department
Center for Research in Electro-Optics and Lasers
Orlando, Florida 32816

Abstract. A method is presented for synthesis of the spectral dependence of infrared sources. Blackbody sources of different temperatures are used as basis functions in this synthesis. The basis functions are linearly combined with appropriate weighting coefficients to approximate the desired spectral distribution. We demonstrate the method by synthesizing blackbody functions of certain temperatures that are not one of the basis functions. The parameters to be determined in the synthesis are the temperatures of the basis functions and the values of the weighting coefficients. Realistic hardware constraints are imposed, such as quantization of the weighting coefficients and the use of a limited number of basis functions. The error in the synthesis is determined as a function of the number of basis sources. The use of four sources was found to be a good compromise, with approximately a 4% error over the 3 to 5 μm band.

Subject terms: infrared scene generation; blackbodies; infrared source simulation; infrared system testing; spectral signature.


CONTENTS
1. Introduction
2. Determination of weighting coefficients
3. Error criterion
4. Choice of basis functions
5. Results
6. Conclusions
7. Acknowledgments
8. References

1. INTRODUCTION

Infrared scene simulation is important for realistic performance testing of thermal imaging systems. The current generation of these systems usually operates on the total flux received within some wavelength passband. More sophisticated systems that are designed to have some immunity to countermeasures such as camouflage, decoys, etc., respond to the distribution of flux over a wavelength range. The added discrimination of these systems arises from their ability to make decisions based on the information contained in the spectral signature or the distribution of flux over wavelength of the target. Testing of these new-generation thermal imaging systems will depend on the accuracy with which an infrared scene generator simulates a desired spectral signature.

Due to practical considerations, IR scene generators required to accurately simulate the spectral distribution of flux would necessarily consist of a finite number of blackbody (BB) sources. This suggests the use of a weighted combination of BB sources to simulate the spectrum of any arbitrary temperature BB (or other thermal source) within a given range. The BB sources would make up a set of basis functions from which the synthesis is made. Since only a finite set of basis functions are used, there would be some error incurred in the simulation. The basic question addressed in this paper is, How many separate thermal sources of different temperatures are required to cover a specific temperature range to a given degree of accuracy?

The hardware implementation of such an IR scene generation system might be envisioned as consisting of a number of BB sources at different temperatures, each with a set of interchangeable neutral density filters. The filters would act as weighting coefficients for each source. The total flux emitted by the system would be the sum of flux from all of the sources. A practical constraint for this system would be to limit the number of filters by quantizing their transmittances in increments of 10%. A constraint of this sort would be necessary because an unlimited number of accurate filters would not be available for implementation in a real-time system. Even the use of circularly variable filters would have a similar type of constraint on the weighting coefficients, due to such factors as positioning accuracy within a short amount of time.

In our model, the weighting coefficients are limited to the range from 0 to 1 in transmittance and occur in steps of 0.1. The limited choice of weighting coefficients affects the accuracy with which a simulation can be made, and it will be shown that this is a major source of error.

The parameters to be determined in our model are the temperatures of the basis functions and the values of the weighting coefficients. The error in the simulation is determined as a function of the number of basis functions for up to nine sources chosen to span a range of temperatures appropriate to the 3 to 5 μm band.

2. DETERMINATION OF WEIGHTING COEFFICIENTS

The method used in choosing the weighting coefficients was to use the two BB basis functions closest in temperature to the source temperature desired in the simulation. The total number of basis functions available will affect the spacing (difference in temperature) of the basis functions, but only two sources will
contribute to any particular synthesis. We have two degrees of freedom (the coefficients) and two constraints (the zeroth and the first moments of the desired radiation function, calculated over the passband of interest). Thus, the values of the two coefficients are found by forming a linear combination of the two basis functions and then matching the zeroth and first moments of this function with those of the desired radiation function as closely as possible.

Let \( T_s \) be the temperature of the BB function to be synthesized, \( T_1 \) and \( T_2 \) be the temperatures of the basis function BB sources used, and \( \varepsilon_1 \) and \( \varepsilon_2 \) be the weighting coefficients to be determined. Using the spectral exitance of a BB given by the Planck equation

\[
M_\lambda(\lambda, T) = \frac{2\pi \hbar c^2}{\lambda^5 [\exp(hc/\lambda kT) - 1]} \text{ W/cm}^2\text{um}^{-1},
\]

we equate the zeroth moments of the BB function to be synthesized and of the linear combination of basis functions to yield

\[
\int_{\lambda_1}^{\lambda_2} M(\lambda, T_s) d\lambda = \int_{\lambda_1}^{\lambda_2} [\varepsilon_1 M(\lambda, T_1) + \varepsilon_2 M(\lambda, T_2)] d\lambda.
\] (2)

Equating the first moments gives the second equation

\[
\int_{\lambda_1}^{\lambda_2} \lambda M(\lambda, T_s) d\lambda = \int_{\lambda_1}^{\lambda_2} \lambda [\varepsilon_1 M(\lambda, T_1) + \varepsilon_2 M(\lambda, T_2)] d\lambda.
\] (3)

For a particular choice of passband, the above two equations can be solved simultaneously to yield solutions for \( \varepsilon_1 \) and \( \varepsilon_2 \). Since the constraint was placed on the filter transmittances that they be quantized in steps of 0.1, the weighting coefficients were then rounded off to one decimal place. Solutions for the weighting coefficients can be obtained for any \( T_s \) between \( T_1 \) and \( T_2 \) using this method. This method of choosing coefficients minimizes the error between the synthesized function and the desired radiation function.

It was found in the analysis that in cases in which the temperatures of the basis sources are spaced widely apart and the desired BB temperature was close to the lower basis function, the algorithm given in Eqs. (2) and (3) for choosing the weighting coefficients gave values greater than 1 for the coefficient of the lower temperature basis source. Since emissivity is defined to be less than 1, the weighting coefficients are not allowed to be greater than 1. In these cases, that coefficient is set equal to 1 and the other is calculated on the basis of the zeroth moment.

Also, a solution using three degrees of freedom (three sources and zeroth through second moments) was attempted and found not to be fruitful. In that case, the coefficients obtained from a simultaneous solution of three resulting equations were often nonphysical, i.e., greater than 1 or negative.

Thus, one part of the problem is solved, that is, How does one choose the coefficients, given the temperatures of the basis functions? The selection of the best basis functions is the subject of Sec. 4, after a suitable error criterion is selected in the next section.

3. ERROR CRITERION

The type of measure chosen to reflect the error between the synthesized and desired exitances should be a realistic description of how well the functions match. A mean absolute error (MAE) over the wavelength region of interest is given by

\[
\text{MAE}(T_s) = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} |M(\lambda, T_s) - M_{\text{syn}}(\lambda)| d\lambda,
\] (4)

where \( T_s \) is the temperature of the BB that is desired in the synthesis and where

\[
M_{\text{syn}}(\lambda) = \varepsilon_1 M(\lambda, T_1) + \varepsilon_2 M(\lambda, T_2)
\] (5)

is the spectral exitance function resulting from the synthesis. The use of MAE can be misleading because of the rate at which the magnitude of the spectral exitance function increases with temperature. According to the Stefan-Boltzmann law, the total exitance of a blackbody increases at a rate proportional to \( T^4 \). This means that a given amount of MAE will be much worse, in terms of a functional mismatch, at lower temperatures than it would be at higher temperatures. To make comparisons possible over a wide range of temperatures, a relative error measure was chosen. The integrated difference between the two functions is normalized to the area under the desired exitance curve. We define the mean relative error (MRE) by

\[
\text{MRE}(T_s) = \frac{\int_{\lambda_1}^{\lambda_2} |M(\lambda, T_s) - M_{\text{syn}}(\lambda)| d\lambda}{\int_{\lambda_1}^{\lambda_2} M(\lambda, T_s) d\lambda}.
\] (6)

4. CHOICE OF BASIS FUNCTIONS

The 3 to 5 \( \mu \text{m} \) range was chosen as the region of interest for the synthesis. Thus, the overall temperature range of the blackbody sources to be synthesized is chosen by considering those temperatures that have a peak in that region. This temperature range is approximately from 550 to 1000 K. These were chosen to be low and high basis temperatures.

With the coefficients chosen in each case to minimize the MRE, the choice of intermediate basis function temperatures was determined from the criterion of minimizing the variance of the MRE in the synthesis over the range of temperatures of interest. This is desirable since it will produce the most consistent synthesis over temperature.

Intuitively, as the spacing between basis temperatures increases one would expect a larger error to be incurred in the synthesis. This is borne out by numerical results in the next section. For a given number of basis sources, there should be an optimum choice for the spacing of these sources in temperature that produces a minimum variance of the MRE. An equal spacing in temperature of the basis sources represents one choice, but it will be shown not to be the optimum choice in this sense.

To begin the process of choosing basis functions, we note that the integrated spectral exitance calculated over a particular \( \Delta \lambda \) passband of interest gives a measure of the effective magnitude of any given basis function. Carrying out the integration in Eq. (2) gives

\[
M(T_s) = \varepsilon_1 M(T_1) + \varepsilon_2 M(T_2),
\] (7)

where the \( M \) quantities represent integrated exitances, for example,

\[
M(T_s) = \int_{\lambda_1}^{\lambda_2} M(\lambda, T_s) d\lambda.
\] (8)
Recall that any particular source to be synthesized is formed by a linear combination of the two basis sources that are closest to it in temperature. Because of the quantization of the coefficients, the error in the synthesis will depend on the relative magnitudes of the two basis functions. The integrated exitances of the two basis functions will differ by some multiplication factor \( \delta \):

\[
M(T_2) = \delta M(T_1),
\]

where \( \delta > 1 \) for \( T_2 > T_1 \). Substituting Eq. (9) into Eq. (7) gives

\[
M(T_j) = M(T_1)(e_1 + \delta e_2).
\]

In the above equation the weighting coefficients are exact quantities, which must be rounded off to one decimal place to conform to the hardware constraints given in the problem. The more the two basis functions differ in integrated exitance, the greater the multiplication factor \( \delta \) will be, and thus the round-off error will increase.

Choosing the integrated exitance of each basis source to be a given percentage of that of the next higher temperature basis source gives equivalent conditions for synthesis between each pair of basis sources. This can be seen by choosing two new basis source temperatures, \( T_1' \) and \( T_2' \), such that \( T_1 < T_2 < T_1' < T_2' \). Using the same \( \delta \) for the multiplication factor gives

\[
M(T_2') = \delta M(T_1').
\]

Suppose we choose a new temperature source, \( T'_j \), to be synthesized such that

\[
\frac{M(T_j)}{M(T_1)} = \text{constant}.
\]

Substituting Eq. (12) into Eq. (10) gives

\[
M(T'_j) = M(T_1)(e_1 + \delta e_2).
\]

By choosing the appropriate basis sources, Eqs. (10) and (13) show that the equivalent conditions exist between the two cases and that identical weighting coefficients will be used in the simulation. Thus, the temperatures of the basis sources should be picked so that their integrated spectral exitances occur in a geometric sequence. We will see in the next section that this method for choosing the basis functions indeed produces a lower variance in the MRE as compared with equally spaced sources.

5. RESULTS

Spectral simulations over the 3 to 5 \( \mu \)m band were performed using from three to nine basis sources spanning the temperature range from 550 to 1000 K. A typical simulation is shown in Fig. 1, which shows the spectra of the actual (desired) and synthesized sources over 3 to 5 \( \mu \)m. In this case the temperatures of the basis sources used were 622 and 714 K and the temperature to be synthesized was 666 K. The MRE for this particular case, with a \( \Delta \lambda \) of 3 to 5 \( \mu \)m, was 0.0556.

For certain applications, it is desirable to choose a smaller \( \Delta \lambda \) for calculation of the integrated exitance. One can specify the relative error (RE) as a function of wavelength:

\[
\text{RE}(\lambda) = \frac{M(\lambda, T_j) - M_{\text{syn}}(\lambda)}{M(\lambda, T_j)}.
\]

which is shown in Fig. 2 for the same synthesis as Fig. 1. The function RE(\( \lambda \)) may be integrated over such \( \Delta \lambda \) as are appropriate for a given application to yield an MRE as given in Eq. (6). We have chosen to plot the MRE curves in this paper with a \( \Delta \lambda \) of 3 to 5 \( \mu \)m. In general, it will be easier to achieve a spectral match over a narrower \( \Delta \lambda \), so we are essentially plotting the worst-case results for our method.

Figures 3 through 9 illustrate how the MRE varies as the temperature to be synthesized varies between the basis temperatures. Note that the RE is zero at the basis temperatures. In all of these cases, the temperatures of the basis sources were chosen such that their integrated exitances formed a geometric sequence between 550 and 1000 K. For comparison, we have plotted in
Fig. 3. MRE using three basis sources (550, 714, and 1000 K), chosen using the geometric sequence technique. The MRE for any particular temperature is the average of the RE over the 3 to 5 μm band.

Fig. 4. MRE over 3 to 5 μm band using four basis sources (550, 650, 790, and 1000 K), chosen using the geometric sequence technique.

Fig. 5. MRE over 3 to 5 μm band using five basis sources (550, 622, 714, 835, and 1000 K), chosen using the geometric sequence technique.

Fig. 6. MRE over 3 to 5 μm band using six basis sources (550, 606, 674, 758, 864, and 1000 K), chosen using the geometric sequence technique.

Fig. 7. MRE over 3 to 5 μm band using seven basis sources (550, 696, 650, 714, 790, 884, and 1000 K), chosen using the geometric sequence technique.

Fig. 8. MRE over 3 to 5 μm band using eight basis sources (550, 589, 634, 685, 745, 815, 901, and 1000 K), chosen using the geometric sequence technique.
SYNTHESIS OF INFRARED SPECTRAL SIGNATURES

Fig. 9. MRE over 3 to 5 μm band using nine basis sources (550, 584, 622, 665, 714, 770, 835, 910, and 1000 K), chosen using the geometric sequence technique.

Fig. 10. MRE over 3 to 5 μm band using six basis sources evenly spaced in temperature (550, 640, 730, 820, 910, and 1000 K).

Fig. 11. MRE over 3 to 5 μm band using seven basis sources evenly spaced in temperature (550, 625, 700, 775, 850, 925, and 1000 K).

Fig. 12. MRE over 3 to 5 μm band using eight basis sources evenly spaced in temperature (550, 614, 679, 742, 807, 871, 936, and 1000 K).

Fig. 13. MRE averaged over 550 to 1000 K vs number of basis sources used (geometric sequence technique). Each point represents the average over temperature of the error data presented in Figs. 3 through 9.

Figs. 10 through 12 a few curves of MRE versus synthesis temperature for equally spaced basis temperatures.

Qualitative comparisons of the plots of MRE in Figs. 3 through 9 and Figs. 10 through 12 indicate that the MRE, when averaged over all synthesis temperatures, tends to decrease as the number of basis sources used increases. Let us define MRE, an MRE averaged over the 550 to 1000 K temperature range, as follows:

\[
MRE = \frac{1}{450 \int_{550}^{1000} \text{MRE}(T_s) dT_s}.
\]

(15)

Figure 13 shows a plot of MRE versus the number of basis sources (data of Figs. 3 through 9, geometric sequence method). This relationship quantifies what one would intuitively suspect, i.e., that the error in simulation should decrease as the number of basis sources increases. The use of four basis sources seems to provide a good compromise between synthesis accuracy and hardware complexity with an MRE of ≈4%.

From the plots, the MRE is found to exhibit oscillatory behavior as the synthesis temperature varies. This results from the
TABLE I. Comparison of methods for basis temperature choice.

<table>
<thead>
<tr>
<th>Number of Sources Used</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geom. Seq. Method</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRE</td>
<td>.0536</td>
<td>.0407</td>
<td>.0451</td>
<td>.0413</td>
<td>.0335</td>
<td>.0276</td>
<td>.0221</td>
</tr>
<tr>
<td>Std. Dev. of MRE</td>
<td>.0418</td>
<td>.0300</td>
<td>.0253</td>
<td>.0230</td>
<td>.0257</td>
<td>.0272</td>
<td>.0255</td>
</tr>
<tr>
<td>Evenly Spaced Temps.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRE</td>
<td>.0590</td>
<td>.0451</td>
<td>.0421</td>
<td>.0352</td>
<td>.0307</td>
<td>.0277</td>
<td>.0239</td>
</tr>
<tr>
<td>Std. Dev. of MRE</td>
<td>.0481</td>
<td>.0324</td>
<td>.0282</td>
<td>.0280</td>
<td>.0273</td>
<td>.0273</td>
<td>.0271</td>
</tr>
</tbody>
</table>

round-off of the weighting coefficients to one decimal place. More than one synthesis temperature may use the same coefficients as a result of the rounding, but only one particular temperature will give the lowest error. As the synthesis temperature changes, the coefficients change and error will oscillate as each set of coefficients gives rise to one particular error minimum.

Thus, it is of interest to compare data on the standard deviation of the MRE for the geometric sequence method and the equally spaced temperature method of choosing basis functions. Performing this comparison, we find that the variance in the MRE is reduced when the geometric sequence method of choosing basis sources is used. We summarize these results in Table I, which is a comparison of MRE and the standard deviation of MRE for basis sources chosen using the geometric sequence of integrated exitance approach and for those chosen using equally spaced temperatures. While the MRE for the geometric sequence approach is slightly larger for three of the seven cases, the standard deviation is always less than that obtained with evenly spaced basis temperatures. The method having the lower standard deviation will produce a more consistent synthesis over temperature.

In Figs. 3 through 9, the overall pattern of the relative error tends to repeat itself between each pair of basis sources. The reason for this is that by choosing the integrated exitances of the basis sources to follow a geometric sequence, the same proportions exist between each pair of basis sources, which gives rise to a similar dependence in each case, as seen in Eqs. (10) and (13). To clarify this, assume that the synthesis temperature we wish to synthesize lies somewhere between the temperatures of two basis sources. The synthesized function will have an integrated exitance given by a linear combination of some fraction of the integrated exitances of the two basis sources. If the next two higher basis sources are used to simulate a source temperature that has the same integrated exitance relative to that of the basis sources, then the same coefficients will be needed to perform this new synthesis. To see the effect that this has on the MRE curve, we recast Eq. (6) as

\[
MRE(T_s) = \frac{\int_{\lambda_1}^{\lambda_2} [M(\lambda, T_s) - (e_1^r M(\lambda, T_1) + e_2^r M(\lambda, T_2))] d\lambda}{\int_{\lambda_1}^{\lambda_2} M(\lambda, T_s) d\lambda},
\]

where the superscript \( r \) on the \( e \) represents rounded values. Moving the absolute values outside the integral gives

\[
MRE(T_s) \Rightarrow \frac{\left| \int_{\lambda_1}^{\lambda_2} M(\lambda, T_s) d\lambda \right|}{\left| \int_{\lambda_1}^{\lambda_2} M(\lambda, T_s) d\lambda \right|}.
\]

Carrying out the integrations in the above equation yields

\[
MRE(T_s) \Rightarrow \frac{\left| M(T_s) - (e_1^r M(T_1) + e_2^r M(T_2)) \right|}{M(T_s)}.
\]

Using Eqs. (9) and (10), we obtain a lower limit on the MRE, which depends only on the exact and rounded values of \( e_1 \) and \( e_2 \):

\[
MRE(T_s) \Rightarrow 1 - \frac{\varepsilon_1 + \delta \varepsilon_2}{\varepsilon_1 + \delta \varepsilon_2}.
\]

If two different basis sources \( T_1 \) and \( T_2 \) from the same geometric sequence are used and a new temperature source \( T_s \) is chosen according to Eq. (12), then the lower limit of the MRE will also be given by Eq. (19). Thus, the general pattern of MRE will repeat itself between each pair of basis sources chosen by the geometric sequence method.

A comparison of MRE approximated by Eq. (19) and that calculated by the exact expression of Eq. (16) is shown in Table II. The data presented there are taken from the simulation that used six basis sources chosen using the geometric sequence technique. The sources that contributed were 606 and 674 K, with a \( \delta \) factor of 1.8305. The MRE values given by Eq. (19) closely match the actual values of MRE.

6. CONCLUSIONS

A method was presented for synthesis of spectral signatures of infrared sources. The synthesis was accomplished using a weighted linear combination of blackbody sources of different temperatures. The weighting coefficients were chosen on the basis of a match of zeroth and first moments between the desired and
TABLE II. Comparison of MRE calculated using Eq. (19) and the actual MRE.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>$\epsilon_1$</td>
<td>$\epsilon_2$</td>
<td>MRE</td>
</tr>
<tr>
<td>606</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>608</td>
<td>0.9869</td>
<td>0.0179</td>
<td>0.0194</td>
</tr>
<tr>
<td>610</td>
<td>0.9731</td>
<td>0.0365</td>
<td>0.0383</td>
</tr>
<tr>
<td>612</td>
<td>0.9583</td>
<td>0.0556</td>
<td>0.1160</td>
</tr>
<tr>
<td>614</td>
<td>0.9427</td>
<td>0.0754</td>
<td>0.0022</td>
</tr>
<tr>
<td>616</td>
<td>0.9262</td>
<td>0.0958</td>
<td>0.0168</td>
</tr>
<tr>
<td>618</td>
<td>0.9089</td>
<td>0.1168</td>
<td>0.0353</td>
</tr>
<tr>
<td>620</td>
<td>0.8906</td>
<td>0.1385</td>
<td>0.0533</td>
</tr>
<tr>
<td>622</td>
<td>0.8714</td>
<td>0.1608</td>
<td>0.0861</td>
</tr>
<tr>
<td>624</td>
<td>0.8512</td>
<td>0.1838</td>
<td>0.0660</td>
</tr>
</tbody>
</table>

The synthesized spectral signature. Two possibilities for the choice of basis functions were investigated. A technique using a geometric sequence of integrated exitance produced a more consistent synthesis over temperature (lower variance of the error) than did the use of basis functions that were equally spaced in temperature.

The realistic hardware constraint of quantization of the weighting coefficients had a large effect on the synthesis accuracy because of coefficient round-off errors. The error in the synthesis was determined as a function of the number of basis sources. It was generally found that the closer the spacing in temperature of the basis sources, the lower the error in the simulation. The use of four sources was found to be a good compromise, with approximately a 4% error over the 3 to 5 $\mu$m band.

The method was demonstrated in the 3 to 5 $\mu$m band by synthesizing blackbody functions of certain temperatures that were not one of the basis functions. The same general technique could also be applied to other passbands or could be used for the synthesis of nonblackbody signatures.

7. ACKNOWLEDGMENTS

This work was supported by the Institute for Simulation and Training of the University of Central Florida and by the Florida High Technology and Industry Council.

8. REFERENCES


Kenneth J. Barnard received the BS degree in electrical engineering from the University of Michigan in 1986 and is currently pursuing a Ph.D. degree in electrical engineering at the University of Central Florida (UCF). He previously held a technical staff position at General Dynamics Land Systems in Troy, Mich. Mr. Barnard is a graduate research assistant at the Center for Research in Electro-Optics and Lasers at UCF. He is a member of SPIE.

Glenn D. Boreman received the BS degree in optics from the University of Rochester in 1978 and the Ph.D. degree in optical sciences from the University of Arizona in 1984. He has held visiting technical staff positions at ITT, Texas Instruments, U.S. Army Night Vision Laboratory, and McDonnell Douglas. Dr. Boreman has been at the University of Central Florida since 1984 and is currently an associate professor of electrical engineering. He is a member of OSA, IEEE, SPIE, and SPSE and currently serves as the Florida section president of OSA.