DUAL-CARRIER TRANSPORT MODEL OF SPRITE DETECTORS

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(Received 12 April 1995)

Abstract—A numerical model for signal-processing-in-the-element (SPRITE) detectors is developed that incorporates both hole and electron motion, the effects of space charge and varying field, and boundary conditions. The model is used to generate a spatial-frequency response for a rectangular SPRITE structure. Further, we use this model to investigate two improved SPRITE structures: the tapered detector and the modulation-doped detector.

1. INTRODUCTION

The signal-processing-in-the-element (SPRITE) detector is widely used in infra-red imaging applications. These detectors work by utilizing the drift motion of the photogenerated carriers to transfer and delay the detected signal in space and time, respectively. This concept has been applied to create monolithic time-delay-and-integration (TDI) detectors, eliminating most of the circuitry normally required for the implementation of such a system.

The basis for the operation of SPRITE detectors is the constant drift velocity of the minority carriers. The drift velocity depends on the effective mobility of these carriers and the electric field impressed on the semiconductor. While linear analytic models [1, 2] can be derived when both of these quantities are assumed constant, they in fact vary over the extent of the detector. This variation of drift velocity results in the degradation of the modulation transfer function (MTF) of the detector.

This effect was first analyzed using an ambipolar approximation to reduce the consideration to only the minority carriers [3]. Through consideration of the effect of excess carriers on the mobility and the electric field, the width of the detector was varied in the along-scan direction such that the change in cross-sectional area exactly compensated for the change in both field and mobility. Detectors were fabricated with their widths tapered according to this analysis, and measurement of their MTF showed improvement over the non-tapered devices [4]. While this approach seems adequate from a design perspective, it does not allow a complete theoretical prediction of the effect of background-induced variations. More recent analyses have included the variation of generation effect and have used numerical integration.
to study the behavior of devices of arbitrary length and taper[5,6]. This more complete model was used to compute the responsivity and revealed an intrinsic saturation in output signal in SPRITE detectors[7].

In this paper, the analysis presented shall improve on the previous work in several ways. First, and most fundamentally, the ambipolar approximation will not be used. While this is unprecedented in SPRITE literature, this dual carrier electrostatic semiconductor model will be more flexible and accurate than previous models, and will allow the investigation of new structure designs. Second, the signal response will be computed using a scanned impulse input. The resulting output will be Fourier analyzed to produce the MTF response of the detector. This is more in keeping with the device's imaging function. In Section 2, the theoretical basis of the model used in this paper will be described. Section 3 will explain the details of the numerical implementation used. The results for rectangular and tapered SPRITEs will be given in Section 4. A new type of structure, the modulation-doped SPRITE, will be introduced in Section 5. Finally, Section 6 will contain the conclusions we can draw from this work.

2. THEORETICAL BASIS

All the previous literature has utilized the ambipolar approximation in the analysis of SPRITE detectors. As the lifetime and background doping level of HgCdTe are improved, the ambipolar description becomes less adequate. The approach taken in this paper will be to use a dual-carrier electrostatic model of electrical behavior in the SPRITE semiconductor. The hole and electron densities, denoted \( p \) and \( n \), respectively, are determined by two interrelated continuity equations:

\[
\frac{dp}{dt} = - \frac{\nabla \cdot J_p}{q} - \frac{pn}{\tau_b} + G,
\]

\[
\frac{dn}{dt} = - \frac{\nabla \cdot J_n}{q} + \frac{pn}{\tau_b} + G.
\]

(1)

The current densities are driven by diffusion and drift:

\[
J_p = - qD_p \nabla p + qE\mu_p, \quad J_n = qD_n \nabla n + qE\mu_n.
\]

(2)

Finally, Gauss's law:

\[
e\nabla \cdot E = q[p + p - N - n],
\]

(3)

is considered, where \( e \) is the permeability, and \( P \) and \( N \) are the positive (donor) and negative (acceptor) dopant concentrations. It should be noted here that the lifetime used in eqns (1), \( \tau_b \), is the bimolecular (two-body) recombination time, which is related to the minority lifetime, \( \tau_m \), by the relation \( \tau_b = P\tau_m \).

SPRITE structures typically have high length-to-width and length-to-thickness ratios, and can be conveniently described with one-dimensional models. In terms of numerical complexity, such models are simpler and faster. However, the effect of device shape and taper are important to this analysis, and these features are markedly two- or even three-dimensional. If it is assumed that the SPRITE is a rectangular solid with only a slowly varying width, then a modified one-dimensional set of equations can be written to describe the device.

To do this, it is convenient to write the equations in the form where the current density and electric field are integrated over the transverse directions. We let the \( z \) axis be parallel to the long axis of the SPRITE. Assuming that the current densities and electric field are constant over any individual cross-sectional area of the device, \( A(z) \), then:

\[
P'(z) = J_p(z)A(z), \quad I'(z) = J_n(z)A(z),
\]

\[
\Phi(z) = E(z)A(z),
\]

(4)

where \( J_p, J_n \) are the hole and electron currents, and \( \Phi \) is the electric flux. Equations (1)–(4) are the basis for the model.

3. NUMERICAL METHOD

Given the above set of equations, the differentials must be discretized so that they can be evaluated. We define normalized variables to preserve the accuracy of the numerical computation. This normalization also aids in the interpretation of the input parameters and the output results of the model.

The discretization of the equations can most easily be viewed by considering the detector to be divided into \( m \) segments, as pictured in Fig. 1 where \( m = 12 \) for illustrative purposes (\( m = 100 \) was actually used). Each segment has an equal thickness, \( \Delta z \), along \( z \). We number these elements from \( 1 \) to \( m \). These elements are bounded by planes, numbered \( 0 \) to \( m \), each perpendicular to the \( z \) axis. Each element is characterized by a volume, \( V_i \), and the densities of the holes, electrons, donors, and acceptors are denoted \( p_i, n_i, P_i, N_i \), respectively. The boundary planes are described by an area, \( A_i \), the hole current, \( I'_i \), the electron current, \( I'_i \), and the electric flux \( \Phi_i \).

Fig. 1. Diagram of SPRITE detector divided into segments, each having a volume and densities of holes, electrons, acceptors and donors. The boundaries between the segments have an area, hole and electron currents, and flux associated with them.
The basic eqns (1)-(4) can be written in terms of these discrete variables by approximating the z derivatives involved by their finite-difference equivalents. This produces:

\[
\frac{dp_i}{dt} = \frac{I_{i+1} - I_i}{qV_i} - \frac{p_i n_i}{\tau_n} + G, \\
\frac{dn_i}{dt} = \frac{I_{i+1} - I_i}{qV_i} - \frac{p_i n_i}{\tau_n} + G, \\
I_i^o = qA_i D_i \frac{p_i - p_{i+1}}{\Delta z} + q\Phi_i \mu_p \frac{p_i + p_{i+1}}{2}, \\
I_i^s = qA_i D_n \frac{n_{i+1} - n_i}{\Delta z} + q\Phi_i \mu_n \frac{n_i + n_{i+1}}{2}, \\
\Phi_i - \Phi_{i-1} = \frac{q}{\epsilon} \left[ P_i + p_i - N_i - n_i \right].
\]  

(5)  

(6)  

(7)

Normalized variables will be indicated by the prime (′) notation. The independent variables, z and t are mostly normalized to the device length, L, and the minority carrier lifetime, \( \tau_m \), thus:

\[
z' = \frac{z}{L}, \quad t' = \frac{t}{\tau_m}.
\]  

(8)

Because we are interested in studying the effect of device taper, the area of the boundaries and the volumes of the elements will vary over the length of the device. Here, these variables are normalized to \( A_0 \) and \( V_0 \):

\[
A_i' = \frac{A_i}{A_0}, \quad V_i' = \frac{V_i}{V_0},
\]  

(9)

where \( A_0 \) is the area of the zeroth boundary, and \( V_0 \) is defined to be the product \( A_0 D \Delta z \). The carrier and dopant densities are normalized to the concentration of donors in the first segment of the device. This yields:

\[
p_i' = \frac{p_i}{P_i}, \quad n_i' = \frac{n_i}{P_i}, \\
p_i' = \frac{p_i}{P_i}, \quad N_i' = \frac{N_i}{P_i}.
\]  

(10)

The currents are normalized to the bias current applied to the device \( I_0 \). The electric flux is normalized to the value of \( \Phi_0 \) required to establish the bias current. This produces:

\[
I_0' = \frac{I_0}{I_0}, \quad I_i^o = \frac{I_i^o}{I_0}, \quad \Phi_i' = \frac{\Phi_i}{\Phi_0}.
\]  

(11)

We can write eqns (5)-(7) in a dimensionless form:

\[
\Delta p_i' = \frac{\Delta p_i'}{\Delta t'} = \frac{1}{V_i'} \left[ I_i^o + \frac{\Phi_i'}{\Delta z'} - \frac{p_i n_i'}{\tau_n'} + R_m \right], \\
\Delta n_i' = \frac{\Delta n_i'}{\Delta t'} = \frac{1}{V_i'} \left[ I_i^o + \frac{\Phi_i'}{\Delta z'} - \frac{p_i n_i'}{\tau_n'} + R_m \right], \\
\Phi_i' - \Phi_{i-1}' = \frac{q}{\epsilon} \left[ P_i' + p_i' - N_i' - n_i' \right].
\]  

(12)  

(13)  

(14)

The differential of the normalized distance is now given by:

\[
\Delta z' = 1/m.
\]  

(15)

Five dimensionless numbers are introduced that contain all of the parameters of the problem. Each can be thought of as a ratio of two physical properties:

\[
R_Q = \frac{L \mu_m}{qP_0 A_0 L} = \frac{Q_b}{Q_d}, \quad R_d = \frac{G \tau_m}{P_0} = \frac{P_b}{P_d}, \\
R_m = \frac{m_p}{m_n}, \quad R_N = \frac{P_0 m_0 A_0 V_0}{I_0 L} = \frac{U_N}{U_0}, \\
R_n = \frac{qP_0 A_0 L}{F_n e} = \frac{F_0}{F_n}.
\]  

(16)

The charge ratio, \( R_Q \), is the ratio of the bias charge flowing in one minority lifetime, \( Q_b \), and the total dopant charge in the device, \( Q_d \). The generation ratio, \( R_d \), is the ratio of the equilibrium carrier concentration resulting from generation, \( P_0 \), and the dopant concentration, \( P_0 \). The mobility ratio, \( R_m \), is the ratio of the hole and electron mobility. The noise–voltage ratio, \( R_N \), is the ratio of the equivalent-noise voltage, \( U_N \), driving the diffusion process and the normal-bias voltage, \( U_0 \). Finally, the electric-flux ratio, \( R_n \), is the flux that would result from a positive charge equal to the amount of donor dopant present in the device, \( \Phi_0 \), over the flux required to establish the bias current.

The scan velocity \( v_s \) is nominally equal to the speed of the minority carriers:

\[
v_{\text{nominal}} = \mu_p E.
\]  

(17)

Because we are using normalized coordinates, we write the following to obtain the normalized nominal scan velocity, \( v' \):

\[
v'_{\text{nominal}} = \frac{v}{L} = \frac{\tau_n \mu_p E}{L} = R_Q R_m.
\]  

(18)

To obtain the response of any detectors, eqns (18)-(20) must be integrated in time. The choice of the time increment, \( \Delta t' \), is critical for convergence. In this work, the increment used is:

\[
\Delta t' = \frac{1}{m^2 R_Q R_n \bar{z}}.
\]  

(19)

Here, \( m^2 \) helps to insure that the numerical code will converge, because it is of the same order as the second derivative implicit in eqns (12) and (13). The inclusion of the scanning speed further improves stability, and it results in the scanned spot moving \( 1/m \) elements per time increment. The use of a
correction factor, $\bar{z}$, is made necessary by the departure of the ambipolar drift velocity from the nominal value. This factor, $\bar{z}$, is the average normalized ambipolar velocity over the detector length.

To incorporate boundary conditions, the assumption is made that the minority carriers exhibit a fixed recombination velocity, $v_0$, at the ends of the detector. It is also assumed that the detector is biased with a constant current equal to $I_0$. These two assumptions result in eqns (20) and (21) for the normalized hole and electron currents at the boundaries:

$$I_p^0 = -R_C v_0' \quad I_n^0 = 1 - I_p^0.$$  \hspace{1cm} (20)

$$I_m^0 = R_C A_m p_m' \quad I_m^0 = 1 - I_n^0.$$  \hspace{1cm} (21)

The new dimensionless ratio $R_C$ is given by:

$$R_C = \frac{v_0 A_0 q P_0 I_0}{I_0} = I_C,$$

where $I_C$ is the surface current resulting from a minority-carrier concentration equal to the majority-dopant concentration. These relations for the terminal currents ensure that an equal amount of positive and negative charge are always present in the detector volume, which produces the required condition of total charge neutrality.

In all preceding models, a rather ad hoc approach was taken in that it was assumed that the readout voltage was proportional to the total charge contained within the readout region. In this model, the normalized output voltage, $U_{\text{out}}^\prime$, can be computed by simply integrating the electric field over the readout and, given that the readout has $k + 1$ elements, this implies:

$$U_{\text{out}}^\prime = \sum_{i = m - k}^{m} \Phi_i \Delta z_i.$$  \hspace{1cm} (23)

The preceding equations give, in a condensed and dimensionless form, a two-carrier electrostatic model of a quasi-one-dimensional SPRITE detector. In the next section, this model will be used to analyze the performance of given detectors, and also to synthesize new detector structures.

4. COMPUTATIONAL RESULTS

The model was implemented on a 486DX2/66-based desktop computer using a Microsoft FORTRAN compiler. We found that breaking the detector into $m = 100$ elements was sufficient to produce good results. This was confirmed by repeating the calculations with twice as many elements and observing no change in the output. We found that a complete analysis typically takes only 5 min of computation time.

The first task for the new model is the computation of the MTF of a simple non-tapered SPRITE detector. This is done both to test the model and also to observe any new phenomena not predicted by the previous work. Table 1 lists typical dimensionless ratios. These are computed from data drawn from Refs [5, 7]. These numbers will be used throughout the remainder of this paper.

Before responses can be produced, however, the detector must first be initialized to its steady state. The model is run without any inputs other than the constant background. The progress of the charge-equilibration process is monitored by observing the mean-square derivative. Figure 2 is a graph of the resulting electron and hole charge densities. Note the strong influence of the boundaries on the charge densities. This distribution is not simply exponential as predicted by the linear theories[8], however, because the electric field is no longer constant and the charge distribution is distorted.

To obtain the impulse response, a small charge input is scanned across the detector to mimic the process of scanning a small light spot. The input is taken to have a normalized width of $\Delta z'$, and it is moved $\text{Ax} / n$ elements in $\Delta t$ time. Figure 3 is a graph of such an impulse response. The pulse is approximately Gaussian, as predicted by the Green’s function analysis. Two new features can be noted. First, the response is skewed to the right, because of causality and the boundary-blocking effect that delay charge transport and produce the slow exponential tail seen. Second, the response has a second small pulse that occurs just after the input spot begins to scan. This new pulse can be attributed to the voltage transient that must occur at the readout bias terminal that compensates for the new charge deposited near the other bias terminal. This peak is not important because of its small scale.

The voltage temporal response can be fast-Fourier transformed to generate the frequency response of the
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Fig. 3. Computed impulse response of typical detector. The main peak is skewed to the right, and there is a small secondary peak at the beginning of the scan (see inset).

Fig. 4. Comparison of MTF curves computed using the Green's function method and the numerical method. Notice that the numerical solution is considerably lower than its analytical counterpart.

Fig. 5. The local ambipolar velocity ratio for the normal rectangular detector, and cross-sectional area and velocity ratio of the tapered detector.

5. MODULATION-DOPED SPRITE

Tapering is used to obtain optimum performance within the basic SPRITE concept. Even the tapered...
SPRITE suffers from the fundamental resolution limit of diffusion. One possible method to counteract the effect of diffusion is to build into the SPRITE some kind of structure that would curtail the spreading of charge. One method would be to modulate the doping level in each segment so that there would be a regular periodic structure along the length of the detector. Figure 7 shows the doping level for six segments near the middle of the detector. The doping is made to have a saw-tooth profile. This kind of doping variation has several impacts. The conductivity and thus the electric flux induced by the bias current varies from point to point. Second, the majority carriers will redistribute themselves under the influence of diffusion, which will cause the formation of space-charge fields. Third, the large variations in local carrier density will produce variations in the ambipolar mobility. Taken all together, it is suggested that this doping profile might reduce diffusion and improve the MTF.

The steady-state carrier distributions are also shown in Fig. 7 for the same six detector segments. The partial depletion of the highly-doped regions and the consequent over-charging of the lightly-doped regions can be seen, as indicated in Fig. 7 by the dark shading for negative charge and light shading for positive charge. This creates a permanent space-charge field in the structure; specifically, there is a high positive field interface in each cycle of the pattern, as shown. Any holes trying to travel backward across this boundary are impeded, which reduces the overall diffusion rate and improves performance. This can be seen by computing the MTF of the modulation-doped detector, shown in Fig. 6. Again, a small improvement in MTF can be seen. It should be noted that this result was achieved with a relatively simple doping profile. Revision and refinement of the profile might well provide further improvement.

6. CONCLUSIONS

In this work, a new model of carrier transport in SPRITE detectors is developed. This model uses a two-carrier electrostatic formalism to produce a system of discrete numerical equations that describe the motion of carriers in the detector. The model is one-dimensional in scale, but the provision for varying the detector cross-section enables the analysis of nonuniform photoconductor filaments. The steady-state carrier distributions produced show a marked deviation from those predicted by previous linear models. The impulse response derived is asymmetric and exhibits a second peak not explained by the classical theory. The MTF generated from this model predicts a lower spatial resolution than predicted by Green's function analysis and is in closer agreement with the measured data than previous analysis[9].

The model is then used to optimize the shape of a tapered detector. This technique is shown to improve uniformity of the ambipolar velocity and the MTF behavior of the final design. Further, the possibility of modulation-doping of the SPRITE is investigated. This doping reduces the diffusive spreading of carriers within the SPRITE. One possible doping profile is demonstrated to improve the MTF.

Acknowledgement—This work was supported by Westinghouse Electric Corporation, Orlando, Florida.
REFERENCES