Modeling the Magnetic Pickup of an Electric Guitar

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This 5-5-5 Presentation Brought to You By:
Geoffrey Tuttle
Magnetic Pickup:

Using electromagnetic induction, motion of string → electric signal

Example of Faraday’s Law

Distortion generated by the pickup?

New model: accurately predicts the change in a B-Field due to the presence of a wire and accounts for the distortion.
The Magnetic Pickup

Permanent magnet induces magnetism in wire

When wire oscillates, the flux through the coil changes.

Time-dependent flux leads to EMF

\[ \Phi_B = \int \int_S \mathbf{B} \cdot d\mathbf{S} \]

\[ |\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| \]
Theory Behind the Model

Simple Magnet

Solenoid

Magnetically Charged Disks

\[ B = B_0 \frac{1}{r^2} \hat{r} \]

\[ B_z = B_0 \frac{\Delta z}{r^3} \]
Model: Magnetic Field Due to Magnetically Charged Disk

The figure depicts one face of a cylindrical magnet of radius ψ centered at (x, y, z) which can be used to determine the magnetic field created by any simple system of permanent magnets.

The magnetic field projected onto the z-axis at (x’, y’, z’) is given by:

$$B_z(x', y', z') = \int_{0}^{2\pi} \int_{0}^{\psi} \frac{\sigma(z' - z)\rho}{\left[(x' - [x - \rho \cos(\phi)])^2 + (y' - [y - \rho \sin(\phi)])^2 + (z' - z)^2\right]^{3/2}} d\rho \, d\phi$$

Can be used to determine the magnetic field created by any simple system of permanent magnets.
Model: Ferromagnetic Wire

Wire: Series of infinitesimally wide magnets

Strength is linearly proportional to field strength of the permanent magnet at the position of the wire

The magnitude of the magnetic induction field at \((x', y', z')\) of an infinitesimally wide magnetic disk is:

\[
|B_w| = \int_0^{2\pi} \int_0^\psi \frac{\sigma \rho}{\sqrt{[x' - [x - \rho \cos(\phi)]^2 + (y' - [y - \rho \sin(\phi)]^2 + (z' - z)^2}}} d\rho \ d\phi
\]
Putting The Model Together

Induced magnetic field is proportional to the strength of the permanent magnet field

The z-component of the magnetic induction field at \((x_p, y_p, z_p)\) due to a section of wire at \((x', y', z')\) is

\[
B_{w,z} = \gamma |B_w| \frac{(z' - z_p)}{[(x' - x_p)^2 + (y' - y_p)^2 + (z' - z_p)^2]^{3/2}}
\]

\[
B_z = B_0 \frac{\Delta z}{r^3}
\]

Where \(\gamma\) is the constant of proportionality relating the strength of the permanent and induced magnetic fields
Experiments and Modeling

3 Experiments

A) Single Magnet

B) Single Magnet and Wire

C) Multiple Magnets of a Commercially Available Bass Pickup (no wire)

After performing the previous experiments to validate the model, it was used to predict the magnetic field of an oscillating string over a pickup.
A) Single Magnet

Fig. 6. The measured magnetic field due to a single magnet as a function of the height above the center of the magnet. The line represents the predictions of the model.

Fig. 7. The measured magnetic field due to a single magnet as a function of horizontal position. The measurements were made 7.1 ± 0.2 mm above the top disk of the magnet. The line represents the predictions of the model.

Agreement between model and experimental results are excellent, especially considering the simplicity of the model.
B) Single Magnet and Wire

For these measurements, the change of magnetic field was measured due to the addition of a ferromagnetic wire.

Fig. 8. Relative change in the magnetic field due to a horizontal movement of the wire above a single magnet. The magnetic field was measured on the top surface of the magnet; the wire was 0.4 ± 0.1 mm above the magnet. The line represents the predictions of the model.

Fig. 9. Relative change in the magnetic field due to a vertical movement of the wire above a single magnet. The magnetic field was measured on the top surface of the magnet. The line represents the predictions of the model.
C) Multiple Magnets of Bass Pickup

Breakdown between model and experimental results for displacement along z-axis

z-component was calculated close to the plane of the magnets

lateral displacement caused the theoretical z-component of the permanent magnet B-field to be significantly weaker.
D) Modeling the Electric Guitar Pickup

Calculated Magnetic Field

![Graph showing the calculated magnetic field with labeled axes and curves indicating the position of the wire in vertical and horizontal orientations, with time and displacement scales.]
Fig. 14. Time-derivative of curves displayed in Fig. 12. The solid curves refer to the signal generated in the coil when the string is oscillating sinusoidally in the horizontal and vertical directions.
D) Modeling the Electric Guitar Pickup

Power Spectra of Time Derivatives of Magnetic Field Calculations

Fig. 16. Power spectra of curves displayed in Fig. 14. The lines are intended only as a guide to the eye.
Conclusions

B-field of a permanent magnet/ferromagnetic wire system can be modeled as a pair of magnetically charged disks and series of infinitesimally wide cylindrical magnets.

Simple model – accessible and agrees well with experimental measurements.

Non-uniform magnetic field creates significant harmonics for both vertical and horizontal oscillations.

Theory deviates from experimental results near the surface of the magnets and offset from center.

Despite these limitations, this model is applicable to the study of electric guitar pickups, where the strings are always above the plane of the magnets.
Side Note

The actual vibrating string was not tested in this paper

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Fourier Analysis of Guitar String over Magnetic Pickup

A-string, Tuned to 107.66 Hz, 0.288 sec sample