

## Appendix 5.a

### MEASURING IMBALANCE

Calculated exposure rates and imbalance indices presented in the paper employ data on classrooms (denoted  $i$ ), schools ( $j$ ) and counties ( $k$ ).

#### White/Nonwhite Imbalance Index

At the school level the racial imbalance index is defined as:

$$S_k^B = (n_k - E_k^*)/n_k, \quad (\text{A-1})$$

where  $n_j$  is school  $j$ 's nonwhite percentage and the exposure rate of white to nonwhite students in county  $k$  is:

$$E_k^* = \frac{\sum W_j n_j}{\sum W_j}, \quad (\text{A-2})$$

where  $W_j$  is the number of white students in school  $j$ .

These measures can be applied at the classroom level. For most calculations students are divided into white and nonwhite, where  $W_{ij}$  includes white students, in any grade, in school  $j$ , that share classroom  $i$  with a 1st, 4th, 7th, or 10th grader, in a given county. For any county  $k$ , the exposure rate of white students to nonwhite students for a particular grade is:

$$E_k = \frac{\sum W_{ij} n_{ij}}{\sum \sum W_{ij}}, \quad (\text{A-3})$$

Where  $n_{ij}$  is the proportion of non-white students, in any grade in classroom  $i$  with a 1st, 4th, 7th, or 10th grader, over the total number of students in the classroom. We defined classroom  $i$  as a unique occurrence of a class that has at least one 1st, 4th, 7th or 10th grader enrolled. We designed a unique classroom identifier as combination of the section number and the internal course number given to each subject by the school.

This exact exposure rate can be compared to the exposure rate based on school-wide racial composition:

$$E_k^* = \frac{\sum W_j n_j}{\sum W_j}, \quad (\text{A-4})$$

where  $W_j$  includes white students, in any grade, in school  $j$ , that share a classroom with a 1st, 4th, 7th, or 10th grader; and  $n_j$  is its nonwhite percentage. Whereas  $E_k$  gives the racial

composition of the typical white student's classroom,  $E_k^*$  gives the racial composition of that student's *school*. Unless the classrooms in each school are typically balanced at that school's racial composition, this exposure rate will be lower than the exposure rate defined above, using racial composition ( $E_k^*$ ). Thus,

$$E_k \leq E_k^* \leq n_k .$$

Imbalance in county  $k$  is defined as the percentage gap between the maximum exposure rate, which would result from racial balance throughout all schools and classrooms in a district, and annual exposure rate  $E_k$ :

$$S_k = (n_k - E_k)/n_k, \tag{A-5}$$

This imbalance can be decomposed into two components: (1) the portion due to racial disparities at the classroom level, within schools:

$$S_k^w = (E_k^* - E_k)/n_k, \tag{A-6}$$

And (2) the portion due to racial disparities between schools, within a county:

$$S_k^B = (n_k - E_k^*)/n_k, \tag{A-7}$$

Note that  $S_k^B$  is the conventional measure of imbalance, based on school level data alone.

### Economic Imbalance Index

At the school level the economic imbalance index is defined as:

$$S_k^B = (p_k - E_k^*)/p_k, \tag{A-8}$$

where  $p_k$  represents the percentage of students eligible for free lunch in the county and  $E_k^*$  is the exposure rate of students not eligible for free lunch to students eligible for free lunch in county  $k$ . Note this index includes private schools. Private schools are defined as schools not supported primarily by public funds, provide instruction for one or more of grades K-12, and have one or more teachers. We assume private school students are not eligible for free lunch.

Unless it is stated otherwise, and only in the case of economic imbalance, data only include elementary and middle schools based on the majority of grades in a given school level. In the event of ties, the school is classified based on the level of the lowest grade.

Data for number of students eligible for free and reduced price lunch for the school year 2000/01 are not available for the following districts: Alamance, Alexander, Alleghany, Anson, Ashe, Avery, Beaufort, Bertie, and Bladen. These districts were excluded from the calculation of income imbalance in this year.

Data for number of students eligible for free and reduced price lunch for the school year 2005/06 are not available for the following districts: Bertie, Harnett, Hoke, and Transylvania. These districts were excluded from the calculation of income imbalance in this year.

### Other Assumptions

The distribution of students by classroom can result in classrooms with fewer than five students. It is likely that such small classrooms would be especially racially unbalanced (all white or all nonwhite students), which would create distortions in the computation of the within imbalance index. To address this issue, we impose a rule to disregard subjects in 1st, 4th, 7th or 10th grade for which the maximum enrollment is below five students.

Similarly, there are cases in which classrooms can be over 30 students (in any grade). To keep classroom size in line with average regular size of classrooms, we impose a rule that for classes larger than 30 students, within imbalance is computed using only the total number of students in 1st, 4th, 7th or 10th grade, instead of all the students in the classroom.